Maths for Computing
Assignment 2

1. *(3 marks)* Prove that if n is a perfect square, then $n + 2$ is not a perfect square.

2. *(3 marks)* Prove that if a and b integers and $a^2 + b^2$ is even, then $a + b$ is even.

3. *(4 marks)* Prove that for every $n \in \mathbb{Z}, 4$ does not divide $(n^2 + 2)$.

4. *(5 marks)* Suppose $a, b, c \in \mathbb{Z}$. Prove that if $a^2 + b^2 = c^2$, then at least one of a and b must be even.

5. *(5 marks)* Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

6. *(7 marks)* Prove that $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$, for every positive integer n.

7. *(7 marks)* Suppose you begin with a pile of n stones and split this pile into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have r and r stones in them, respectively, you compute rs. Show that no matter how you split the piles, the sum of the products computed at each step equals $n(n-1)/2$.

8. *(8 marks)* At a tennis tournament, there were 2^n participants, where n is a positive integer, and any two of them played against each other exactly one time. Prove that we can find $n + 1$ players that can form a line in which everybody has defeated all the players who are behind him in the line.

9. *(10 marks)* Prove the inequality between the geometric mean and the arithmetic mean using induction, that is, prove that if $a_1, a_2, \ldots, a_n$ are non-negative numbers, then

$$\sqrt[n]{a_1 a_2 \ldots a_n} \leq \frac{a_1 + a_2 + \ldots + a_n}{n}.$$